THE FUNDAMENTAL LAW OF THE GRATING

BY

JANET TUCKER HOWELL

DISSERTATION

SUBMITTED TO THE BOARD OF UNIVERSITY STUDIES OF THE JOHNS HOPKINS UNIVERSITY IN CONFORMITY WITH THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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THE COINCIDENCE METHOD

The fundamental law of the grating is that $N\lambda = w\ (\sin\theta + \sin\phi)$ where N is the order of the spectrum, λ is the wave-length, w is the distance apart of the lines of the grating from center to center, θ is the angle between the incident light and the normal to the grating, and ϕ is the angle between the diffracted light and the normal. With the concave grating mounting used by Rowland $\phi = 0$ and the law of the grating becomes $N\lambda = w \sin\theta$. It follows therefore that, for any one angle θ, $N\lambda$ is constant, and it is possible, knowing standard wave-lengths in one order, to measure the wave-lengths of lines in the superimposed spectra of other orders. This procedure, known as the method of coincidences, was used by Rowland in determining his Table of Standards; and in parts of the spectrum in which standards have not been established by independent interferometer measurements, it is still the only method for the determination of wave-lengths.

SUMMARY OF PREVIOUS WORK ON GRATING MEASUREMENTS

Kayser in his first paper on "Standards of Wave-Length"[1] took up the question of the errors in Rowland's Table of Standards. These errors are due largely to the fact that Rowland shifted different parts of the spectrum to make his solar and arc spectra agree, but Kayser thought they were due partly to a second cause, namely, the unreliability of the coincidence method, and the object of his work was to test this method. Using the method of coincidences Kayser determined the wave-lengths of a group of third-order lines from second-order standards with two different gratings and found a systematic difference in his values amounting to 0.033 A. This could be accounted for by supposing one of the gratings to have suffered a linear displacement of the rulings of one part

[1] *Astrophysical Journal*, 19, 157, 1904.

with respect to the rest,[1] were it not for the fact that in calculating the Fabry and Perot standards by the coincidence method he found an error amounting to 0.019 A even for his better grating. As there was no reason for doubting the values of Fabry and Perot, Kayser came to the conclusion that neither of his gratings was available for coincidence work and thought it highly improbable that gratings existed perfect enough for accurate measurements by this method. He therefore confined his work on wave-length measurements to direct interpolation between Fabry and Perot standards of the same order.[2]

Since Kayser's last paper, work has been done on this subject in several directions. Papenfus in 1911[3] published two papers on the availability of the coincidence method for wave-length measurements. Working with a 13-ft. concave grating having 10,000 lines to the inch he determined the values of the tertiary standards from λ 6065 to λ 6678 by direct interpolation and from λ 4076 to λ 4376 by the coincidence method. He compared his values in both cases with Kayser's values determined by direct interpolation and found the same order of difference, which showed that no appreciable error had been introduced by the use of the coincidence method in the case of his grating.

Goos has done some recent wave-length work with a plane grating having 7000 lines to the inch.[4] He investigated the effect of varying the light-source, using first a short arc of 3 or 4 mm and then a long arc of 8 or 9 mm. With the short arc the lines were broader and when the broadening was unsymmetrical the values found with the two types of arc differed sometimes by as much as 0.02 A. So he arrives at the conclusion that a variation in the light-source used will account for large variations in the values found for tertiary standards and that before these tertiary standards can be established with any degree of accuracy a more exact definition of the standard arc is necessary.

The most recent work on tertiary standards has been done at

[1] *Ibid.*, **18**, 278, 1903. [2] *Ibid.*, **32**, 217, 1910.

[3] *Zeitschrift für wissenschaftliche Photographie*, **9**, 332, 349, 1911.

[4] *Astrophysical Journal*, **35**, 221, 1912; **37**, 48, 1913; **38**, 141, 1913.

Mount Wilson by St. John and Ware.[1] They worked both at Pasadena and at Mount Wilson and found that owing to the difference in pressure amounting to about one-fifth of an atmosphere, due to the height of Mount Wilson, some of the lines were appreciably displaced and unsymmetrically widened. The mean difference for lines suffering a great displacement amounted to about 0.05 A per atmosphere. The secondary standards were displaced only 0.006 A per atmosphere, and in the region investigated they found 25 lines which were symmetrically widened and only slightly displaced under pressure which they recommend as tertiary standards.

The work of Goos on the variation of wave-length with the length of arc and that of St. John and Ware on the displacement and unsymmetrical broadening of some lines under pressure shows that there is a chance for large errors in wave-length measurements if the experimental conditions vary to any extent. It leads one to suspect that, after all, the inadaptability of one or both of Kayser's gratings to coincidence work may not have been altogether responsible for the errors found. In calculating the Fabry and Perot standards by the coincidence method it is possible that, although the standards themselves might not be subject to any appreciable variation in wave-length, the intermediate tertiary standards used might belong to the type that is subject to displacement and unsymmetrical broadening under varying experimental conditions. As these intermediate tertiary standards lie between λ 3550 and λ 3630, a region which has not been investigated in this respect, there is as yet no conclusive evidence on this point.

There is one source of error in coincidence work for which correction should be made in any absolute determinations of wave-length. This error is due to the fact that the dispersion of air varies with the temperature. The following example will show the magnitude of the effect.

$n=\frac{\lambda_e}{\lambda}$ where n is the refractive index, λ_e is the wave-length in ether and λ is the wave-length in air. Suppose that the wave-lengths λ 5640, first order, and λ 2820, second order, coincide exactly at 0° C. and 760 mm pressure.

[1] *Op. cit.*, **36**, 14, 1912.

Then

$$\lambda_{e1} = n \times 5640 = 1.0002924 \times 5640$$

and

$$\lambda_{e2} = n \times 2820 = 1.0003091 \times 2820.$$

At 10° and 760 mm the values for n are different.

$$\lambda_1 = \frac{\lambda_{e1}}{n} = \frac{1.0002924 \times 5640}{1.0002761} = 5640.09187$$

$$\lambda_2 = \frac{\lambda_{e2}}{n} = \frac{1.0003091 \times 2820}{1.0002919} = 2820.04847$$

$$\therefore 2\lambda_2 - \lambda_1 = 5640.0969 - 5640.0919 = 0.005 \text{ A.}$$

So that two lines which coincide exactly at 0° C. will differ by 0.005 A at 10° C. This shows the importance of keeping the temperature constant and may account to a large extent for the variations found in measurements by the coincidence method. In exact determinations of wave-length by this method all values should be reduced to standard conditions.

The work of Papenfus showed that his grating is adapted to coincidence work and it is possible that one of Kayser's gratings may be also, but the coincidence method is far from being reinstated in the position of importance it enjoyed at the time of Rowland's work. It was with the purpose of further testing the method that the piece of work here reported was undertaken.

RESULTS OF THIS INVESTIGATION

The work consists of two parts. In the first part, using the international standards from λ 5266 to λ 5434 in the second order, I determined from them by the method of coincidences the superimposed third-order lines in this region with two gratings, one having 15,000, the other 20,000 lines per inch. This is the same region that Kayser used in comparing his 20,000- and 16,000-line gratings. My results agreed remarkably well and show that the two gratings examined are perfect enough for wave-length determinations by the method of coincidences. But this part of the work did not in itself constitute a thorough proof of the law of gratings. In the second part of my paper I have shown that it is possible, starting with one standard line and assuming the law of

gratings, to determine the value of secondary standard lines to such a degree of accuracy that it would be feasible—if the secondary standards were not already determined by independent methods—to start from one known line and build up a whole system of wave-lengths from it, provided of course that the gratings used in the work were as good as the ones with which I have been working. This second part of my work shows that a system of wave-lengths can be built up from one line by means of the coincidence method which will attain the degree of accuracy originally claimed for this method by Rowland and therefore verifies the fundamental law of gratings.

APPARATUS

After the death of Rowland his two ruling machines were not used until 1910, when they were again put into working order by Dr. J. A. Anderson. Several of the gratings ruled last year by Dr. Anderson have been at my disposal. I have had therefore exceptional opportunities for carrying on grating work. I only regret that I have not had the time to test a larger number of them in the same and in different directions. In the first part of my work, comparing tertiary standards obtained by the coincidence method with two gratings, I used one of Rowland's six-inch concave gratings with a 21-foot radius and 20,000 lines to the inch, and one of Anderson's six-inch concave gratings with approximately the same radius and 15,000 lines to the inch. In the second part of my work it was necessary to work with superimposed spectra of higher orders, so I used one of Anderson's concave gratings having a radius of 21 feet and 7,500 lines per inch.

The grating mounting was that used by Rowland and Jewell in their work. The experimental conditions were kept as constant as possible, so that any line-displacement due to a variation in the light-source would occur to the same extent throughout the work. In the event therefore of a standardization of the light-source to be employed in wave-length measurements, my values could not be used for tertiary standards. They are, however, correct in relation to each other and this is all that is necessary in comparing the values obtained from different gratings in a proof of the coincidence method. The iron arc used was about 4 mm long and was operated at 110 volts with a direct current of between 5.5 and 6.5 amperes.

Owing to the fact that the Johns Hopkins University is situated in the middle of a city, it was found impossible to use plates taken in the daytime, as the vibrations due to the traffic seriously impaired the sharpness of the lines. All the plates used for measurement were therefore taken between 12 P.M. and 6 A.M. The method of measuring the plates was as follows: The secondary standards λ 5266.569 (group *d*) and λ 5434.527 (group *a* of Gale and Adams) were used to determine the scale of each plate in the second order. Using the scale thus determined, the values of the intermediate standard lines λ 5371.495 (group *a*) and λ 5405.78 (group *a*) were calculated. From the differences between the calculated and standard values a calibration-curve of each plate was drawn from

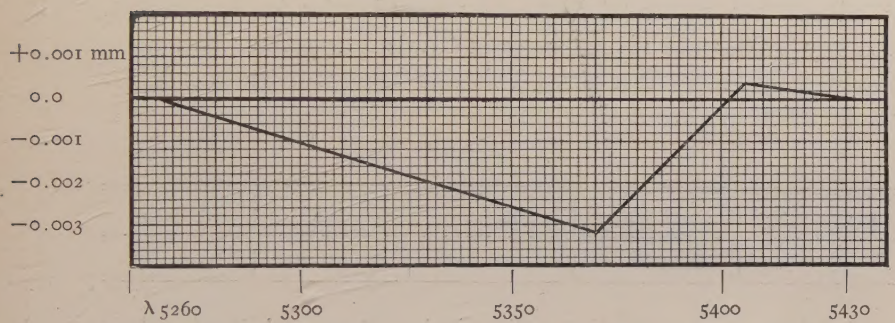

FIG. 1.—Typical calibration-curve for a plate between λ 5266 and λ 5434

which the readings for all other lines were corrected. A typical calibration-curve is shown above in Fig. 1.

The measuring engine was specially constructed for use in this piece of work. The screw was ground by the Rowland method until all periodic errors were sufficiently eliminated for the purpose and it was estimated that the errors outside of the error of the screw did not amount to more than one-fifth of a division on the head of the screw. One division on the screw-head = 0.001 mm and, as it was found impossible to count on successive readings agreeing to within less than 0.005 mm, the error of the instrument is negligible in comparison with the error of setting on a line. The readings were made to thousandths of a millimeter, and from twelve to fifteen readings were made on each line. The plates

were reversed after the whole region had been measured in one direction three times.

The scale of my plates was such that for the 20,000-line grating 1 mm = 0.9702 A and for the 15,000-line grating 1 mm = 1.30729 A, in the second order. After correcting the readings for each line from the calibration-curve of the plate the wave-lengths of the lines were found in terms of this scale in the second order and then reduced to third-order values by multiplying by the factor $\frac{2}{3}$.

FIRST PART—RESULTS

The region between λ 5266 and λ 5434 was measured as described above. In this region there is only one second-order line which is on the list recommended by St. John and Ware as tertiary standards. It was therefore the only second-order line I measured, and my values are given below in Table I, together with those obtained by other observers. This line belongs to group *a* in the classification of Gale and Adams.

TABLE I

20,000 Grating	15,000 Grating	Kayser	Goos	St. John
5429.7005	5429.7001	5429.701	5429.700	5429.702

As there was no way of telling which of the third-order lines were the best for this work and the least affected by varying experimental conditions, I measured all those which were sharp on both sets of plates. The reading for any one line on one plate did not differ from the mean value of the four plates used by more than 0.003 mm. The greatest difference found between the values of any third-order line calculated by the two gratings amounted to 0.005 A, a difference which occurs only once in the 47 lines measured. The sum of the differences amounts to +0.0620 and −0.0237 A, the average difference being only $+\frac{0.0383}{47} = +0.0008$ A and the average variation $= \pm\frac{0.0857}{47} = \pm 0.0018$ A. So the difference between two lines measured on different gratings by the coincidence method is of the same order of magnitude as the

difference between values of the same line measured on different plates. This proves that the two gratings employed were perfect enough to be used in wave-length measurement by the coincidence method. The tabulated results are given in Table II.

TABLE II

20,000 Grating	15,000 Grating	Difference	Kayser	Rowland
3513.8220	.8226	+0.0006	.821	.97
3521.2669	.2674	+ .0005	.26	.41
3524.0803	.0809	+ .0006	.07	.22
3524.2463	.2469	+ .0006	.24	.39
3526.0470	.0453	− .0017	.03	.18
3526.1705	.1721	+ .0016	.23	.38
3526.3843	.3839	− .0004	.38	.53
3526.6785	.6806	+ .0021	.66	.81
3527.7999	.8033	+ .0034	.78	.93
3529.8235	.8265	+ .0030	.80	.95
3533.0115	.0103	− .0012	.00	.15
3533.2028	.2057	+ .0029	.19	.34
3536.5597	.5632	+ .0035	.55	.70
3540.1314	.1349	+ .0035	.12	.27
3541.0925	.0939	+ .0014	.09	.24
3542.0819	.0787	− .0032	.08	.23
3545.6437	.6487	+ .0050	.62	.77
3553.7459	.7432	− .0027	.72	.87
3554.1250	.1238	− .0012	.11	.26
3554.9299	.9265	− .0034	3554.92	3555.07
3556.8822	.8831	+ .0009	3556.880	3557.03
3558.5213	.5211	− .0002	.52	.67
3565.3862	.3880	+ .0018	.38	.53
3570.1049	.1045	− .0004	.12	.27
3571.9985	.9968	− .0017	3572.00	3572.15
3575.3745	.3750	+ .0005	.35	.50
3581.2017	.2024	+ .0007	.20	.35
3582.2069	.2061	− .0008	.19	.34
3584.6657	.6672	+ .0015	.65	.80
3584.9628	.9636	+ .0008	3584.96	3585.11
3585.3255	.3274	+ .0019	.32	.47
3585.7118	.7142	+ .0024	.71	.86
3586.1172	.1165	− .0007	.11	.26
3586.9921	.9918	− .0003	3586.97	3587.12
3589.1115	.1103	− .0012	.10	.25
3594.6419	.6397	− .0022	.62	.77
3603.2095	.2106	+ .0011	.20	.35
3605.4595	.4615	+ .0020	.47	.62
3606.6846	.6865	+ .0019	.682	.83
3608.8645	.8621	− .0024	3608.85	3609.01
3610.1584	.1597	+ .0013	.16	.31
3612.0805	.0851	+ .0046	.08	.23
3617.7899	.7939	+ .0040	.78	.93
3618.3899	.3923	+ .0024	.38	.53
3618.7723	.7730	+ .0007	.77	.92
3621.4634	.4655	+ .0021	.46	.61
3622.0061	.0088	+ .0027	.00	.15

SECOND PART—THEORY

In this part of my work I have tried to furnish a real proof of the accuracy with which the fundamental law of the grating holds. The grating used in this part was one of Anderson's with 7,500 lines to the inch and a radius of 21 feet. For my fundamental line I chose λ 5232.957, one of the international secondary standards (group *d*) and proceeded as follows to determine the scale of my plate, knowing

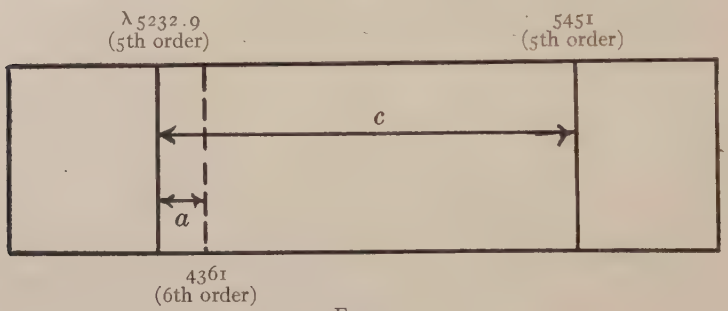

FIG. 2

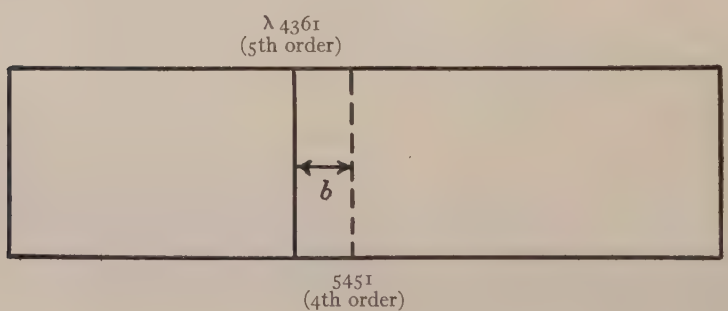

FIG. 3

only this one wave-length and the law of gratings. I first photographed this line in the 5th order. A plate of this type is illustrated in Fig. 2. On Fig. 2 we know the wave-length of line λ 5232.957 in the 5th order. Near it is a line in the 6th order which we shall call 4361, although its wave-length is not known. The distance between λ 5232.957 (5th) and 4361 (6th) is *a* mm. Then, if *x* mm = 1 A in the 5th order,

$$\text{the distance } a = \frac{a}{x} \text{ A.}$$

The wave-length of the line 4361 will then be $\lambda_1=(5232.957+\frac{a}{x})$ A in terms of the 5th order, which gives

$$\lambda_1=\left[5232.957+\frac{a}{x}\right]\frac{5}{6}\text{ A}$$

in the 6th order.

The grating is now shifted so as to bring the line 4361 in the 5th order on the middle of the plate. A plate of this type is illustrated in Fig. 3. At a distance b mm from 4361 in the 5th order there will be a line in the 4th order which we shall call 5451. This line also occurs on Fig. 2 in the 5th order at a distance c mm from λ 5232.957, the standard line:

$$b=\frac{b}{x}\text{ A}$$

in the 5th order

$$\therefore 5451=\lambda_2=\left[5232.957+\frac{a}{x}\right]\frac{5}{6}+\frac{b}{x}\text{A}$$

in the 5th order

$$\lambda_2=\left(\left[5232.957+\frac{a}{x}\right]\frac{5}{6}+\frac{b}{x}\right)\frac{5}{4}\text{ A}$$

in the 4th order.

Also from Fig. 2,

$$\lambda_2=5451=\left(5232.957+\frac{c}{x}\right)\text{A}$$

in the 5th order

$$\therefore \left(5232.957+\frac{c}{x}\right)=\left(\left[5232.957+\frac{a}{x}\right]\frac{5}{6}+\frac{b}{x}\right)\frac{5}{4}$$

$$\therefore 5232.957x=24c-25a-30b$$

$\therefore x=$ no. of mm per A in the 5th order

$$x=\frac{24c-25a-30b}{5232.957}.$$

In practice I used four lines in the neighborhood of λ 5232.9 instead of the one referred to above as 4361. Then for the line given as 5451 I used one of the international standards, λ 5455.614 (group a), so that my scale, obtained entirely by the coincidence method, might more easily be compared with the scale obtained directly from the two standards λ 5232.957 and λ 5455.614. In order to identify the lines it was of course necessary to take plates

with color-filters to cut out all except certain orders. I then took four plates of Fig. 2 and four of Fig. 3. The values for b for each of the four lines used in place of 4361 were then averaged for the four plates of Fig. 3, each line being read nine times. These average values for b were then used with the different values for a and c obtained from each of the four plates of Fig. 2. The values for a and c and the average values of b are given in Table III. The four lines used for 4361 are denoted by the symbols I, II, III, IV. On two plates there were only three lines good enough for accurate measurement.

The values for x, the number of mm per angstrom in the 5th order are given in Table IV. The subscripts I, II, III, IV indicate which of the four lines was used in obtaining x and the mean value of x obtained by the method of coincidences is then compared with x (standard), the value found by assuming the two international secondary standards λ 5232.957 and λ 5455.614 in the usual way.

The greatest error for the scale of the plates determined by this method amounts to one part in a hundred thousand and the mean error to three parts in a million. It is easier, however, to judge the method by comparing the values for the standard line λ 5455.614 as found by the coincidence method with the accepted value as found by independent interferometer measurements. This comparison is made in Table V.

We therefore see that the maximum error introduced by calculating one standard line from another by the method of coincidences is 0.0029 A and the mean error amounts to only 0.0007 A.

In calculating the line λ 5455.6 from the original line λ 5232.9 there has of course been no calibration-correction made. But even in starting with a set of the international standards it is necessary to use two of them without calibration-correction of any kind in order to get the scale of the plate. In order to carry out the scheme of building up a whole system of wave-lengths from one line it would be necessary, after establishing a second line in the manner described above, to determine the intermediate standard lines by coincidences in the same way, starting from each of the two fundamental standards as base line and also from each new

standard as it is determined. In this way, by averaging the results, each line will be established with sufficient accuracy to be used as a standard line and from these standards a calibration-curve may

TABLE III

	I	II	III	IV	
Plate $I_{(a)}$					
a	− 9.2226	+7.7708	+17.3393	+26.0642	
b	+11.2161	−2.9545	−10.9239	−18.1977	
c					212.5546
Plate $I_{(b)}$					
a	− 9.2228	+7.769	+17.342	+26.0616	
b	+11.2161	−2.9545	−10.9239	−18.1977	
c					212.5507
Plate $I_{(c)}$					
a		+7.7761	+17.3419	+26.067	
b		−2.9545	−10.9239	−18.1977	
c					212.550
Plate $I_{(d)}$					
a	− 9.2215		+17.3412	+26.066	
b	+11.2161		−10.9239	−18.1977	
c					212.5534

TABLE IV

Plate	$X_{(I)}$	$X_{(II)}$	$X_{(III)}$	$X_{(IV)}$	X_{mean}	$X_{standard}$	Difference
$I_{(a)}$	0.954607	0.954656	0.954631	0.954649	0.954636	0.954628	−0.000008
$I_{(b)}$	.95459	.954614	.95460	.954614	.954604	.954610	+ .000006
$I_{(c)}$		.954598	.95461	.954614	.954607	.954610	+ .000003
$I_{(d)}$	0.954595		0.954617	0.954618	0.95461	0.95462	+0.00001

TABLE V

Plate	$C_{(mm)}$	$\frac{c}{x_{mean}} = C_A$	$5232.957 + C_A$ St. Line Calc.	Standard I.A.	Difference from St. 5455.614
I_a—	212.5546	222.6553	5455.6123	5455.614	−0.0017
I_b—	212.5507	222.6585	5455.6155	5455.614	+ .0015
I_c—	212.550	222.65707	5455.61407	5455.614	+ .00007
I_d—	212.5534	222.6599	5455.6169	5455.614	+0.0029

be drawn for the plate. Then the other lines of all the superimposed spectra in the region can be determined. These lines will give starting-points for similar work in other parts of the spectrum. Of course as the steps involved in the work carry one farther and

farther from the two original lines, errors might accumulate to a very appreciable extent, but at the same time every new line that is established will give another reference line from which the next lines may be determined. So that by tying up each new line by coincidences with many other lines, some of greater, some of less, wave-length, the errors introduced will offset each other.

My original intention in taking up this work was to build up, in some part at least, a system such as this. To build up an entire system, determining each line with a sufficient number of coincidences to insure the desired accuracy, would be a stupendous task. It would also be an unnecessary one, because the values obtained for the standard lines could not obtain an accuracy greater than that of the international system. It is therefore simpler to use these standards to begin with. I have shown, however, that, having these standards throughout part of the spectrum, it is possible to investigate the region into which they do not extend by means of the coincidence method with an accuracy as great as has been obtained as yet in wave-length determination by direct interpolation. Before a grating can be used for this work it must be tested by the method given in Part II of this paper in order to see how closely it obeys the law of gratings. The results obtained with my three gratings have been so good that it is perhaps not too much to say that a grating which is unfit for coincidence work is the exception and not the rule.

In conclusion I wish to thank Dr. J. S. Ames and Dr. J. A. Anderson, under whose direction this research has been conducted, for the help and advice they have given me and the interest they have taken in my work.

JOHNS HOPKINS UNIVERSITY
June 1913

LIFE

Janet Tucker Howell, the daughter of William Henry and Janet Tucker Howell, was born in Baltimore, Maryland, on January 1, 1889. She received her preparatory school education at the Bryn Mawr School in Baltimore, where she was a student from September 1895, until June 1906. In the autumn of 1906 she entered Bryn Mawr College and was the holder of the Bryn Mawr School scholarship for two years. In June 1910, she received the degree of Bachelor of Arts from Bryn Mawr College. From October 1910, until June 1913, she was a graduate student in physics, astronomy, and mathematics at the Johns Hopkins University. In June 1912, she received the degree of Master of Arts and in June 1913, the degree of Doctor of Philosophy.